THE CELLULAR PARTICLE SWARM OPTIMIZATION ALGORITHM

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ABSTRACT

This work presents a variant of the Particle Swarm Optimization (PSO) original algorithm, the Cellular-PSO. Inspired by the cellular Genetic Algorithm (GA), particles in Cellular-PSO are arranged into a matrix of cells interconnected according to a given topology. Such topology defines particle’s neighborhood, inside which social adaptation may occur. As a consequence, population diversity is increased and the optimization process becomes more efficient and robust. The proposed Cellular-PSO has been applied to the nuclear reactor core design optimization problem and comparative experiments demonstrated that it is superior to the standard PSO.

1. INTRODUCTION

In a nuclear reactor design, a wide variety of optimization problems arise from the need of achieving high efficiency, availability and safety levels. Optimum design parameters are searched in very complex search spaces, under lots of constraints. Therefore, efficiency and robustness of the optimization techniques are quite important.

Several decades ago, traditional gradient-based optimization techniques had been used \cite{1}. Due to the non-linearity, multimodality and, poor knowledge about the search domain of most complex problems, the use of more robust and appropriate techniques such as Simulated Annealing (SA) \cite{2} and Genetic Algorithms (GA) \cite{3-7} have been proposed.

Seeking for enhancements in the optimization processes, other methodologies have been proposed. Among them, Particle Swarm Optimization (PSO) \cite{8}, a population-based metaheuristic has demonstrating some advantages over other techniques. Several applications of PSO to nuclear problems have been recently published in literature \cite{9-12}. In this work, a variant of the standard PSO - the Cellular-PSO - is described and results of its application to a nuclear core design optimization problem demonstrated advantages over the standard PSO.

2. THE CELLULAR PSO

Particle Swarm Optimization (PSO) is an optimization metaheuristic inspired by the behavior of biological swarms and social adaptation. In PSO, a swarm of structures encoding solution
candidates ("particles") “fly” in the n-dimensional search space of the optimization problem looking for optima or near-optima regions. The position of a particle represents a solution candidate itself, while the velocity attribute, provides information about direction and changing rate. Particles are guided by two components: i) cognitive information based on particles’ own experience and ii) social information based on observation of neighbors. Let \( \overline{X}_i(t) = \{x_{i,1}(t),...,x_{i,n}(t)\} \) and \( \overline{V}_i(t) = \{v_{i,1}(t),...,v_{i,n}(t)\} \) be, respectively, the position and the velocity of particle \( i \) in time \( t \), in an n-dimensional search space. Considering that \( \overline{pBest}_i(t) = \{pBest_{i,1}(t),...,pBest_{i,n}(t)\} \) is the best position already found by particle \( i \) until time \( t \) and \( \overline{gBest}_i(t) = \{gBest_{i,1}(t),...,gBest_{i,n}(t)\} \) is the best position already found by a neighbor until \( t \), the PSO updating rules for velocity and position are given by:

\[
\begin{align*}
    v_{i,n}(t+1) &= w v_{i,n}(t) + c_1 r_1 (pBest_{i,n}(t) - x_{i,n}(t)) + c_2 r_2 (gBest_{i,n}(t) - x_{i,n}(t)) \\
    x_{i,n}(t+1) &= x_{i,n}(t) + v_{i,n}(t+1)
\end{align*}
\]

where \( r_1 \) and \( r_2 \) are random numbers between 0 and 1. Coefficients \( c_1 \) and \( c_2 \) are given acceleration constants towards \( pBest \) and \( gBest \) respectively and \( w \) is the inertia weight.

The inertia weight, \( w \), is the responsible for the scope of the exploration of the search space. High values of \( w \) promote global exploration and exploitation, while low values, lead to local search. A common approach to provide balance between global and local search is to linearly decrease \( w \) during the search process.

The swarm is randomly initialized. Then, while stopping criterion is not reached, particles move according velocity and positions equations (eqs. 1 and 2). The PSO algorithm pseudo code can be seen in Figure 1.

```
Algorithm PSO
begin
    for i=1 to n_particles do begin
        randomize(X_i); randomize(V_i);
    end;
    for iter=1 to iter_max do begin
        for i=1 to n_particles do evaluate(X_i);
        for i=1 to n_particles do update(pBest_i,gBest_i);
        for i=1 to n_particles do begin
            V_i = w*V_i + c_1*r_1*(pBest_i-X_i) + c_2*r_2*(gBest-X_i);
            X_i = X_i + V_i;
        end;
    end;
end.
```

**Figure 1. Standard PSO pseudo code.**

The proposed Cellular-PSO is a parallel/distributed approach in which particles are arranged in a 2D-grid, as illustrated in Figure 2. In the Cellular-PSO, \( gBest \) is no more visible to all particles. Instead, particles use a \( lBest \) (local best), which is the best particle among the neighbors. Such neighborhood restriction delays the information exchange between non-neighbor particles, increasing the diversity in the search. As consequence, the method becomes more robust and efficient.
3. THE OPTIMIZATION PROBLEM

It is considered a simplified cylindrical three-enrichment-zone Pressurized Water Reactor (PWR), with typical cell composed by moderator (light water), cladding and fuel. Figure 3 illustrates such reactor. The design parameters as well as their ranges are shown in Table 1.

![Cellular-PSO](image)

**Figure 2. Cellular-PSO**

![The reactor and its typical cell](image)

**Figure 3. (a) The reactor and (b) its typical cell**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel radius (cm)</td>
<td>$R_f$</td>
<td>0.508–1.27</td>
</tr>
<tr>
<td>Cladding thickness (cm)</td>
<td>$\Delta c$</td>
<td>0.0254–0.254</td>
</tr>
<tr>
<td>Moderator thickness (cm)</td>
<td>$R_e$</td>
<td>0.0254–0.762</td>
</tr>
<tr>
<td>Enrichment of zone 1 (%)</td>
<td>$E_1$</td>
<td>2.0–6.0</td>
</tr>
<tr>
<td>Enrichment of zone 2 (%)</td>
<td>$E_2$</td>
<td>2.0–6.0</td>
</tr>
<tr>
<td>Enrichment of zone 3 (%)</td>
<td>$E_3$</td>
<td>2.0–6.0</td>
</tr>
<tr>
<td>Fuel material</td>
<td>$M_f$</td>
<td>${U$-metal or UO$_2}$</td>
</tr>
<tr>
<td>Cladding material</td>
<td>$M_c$</td>
<td>zircaloy-2, aluminum or stainless-304</td>
</tr>
</tbody>
</table>

The objective of the optimization problem is to maximize the average thermal flux, $\phi_{AVE}$, of the proposed reactor, considering as constraints the criticality, the sub-moderation and a maximum peak factor $fp_{MAX}$. The optimization problem can be written as follow:

Minimize:

$$\phi_{AVE}(R_f, \Delta c, R_e, E_1, E_2, E_3, M_f, M_c)$$

Subject to:

$$fp(R_f, \Delta c, R_e, E_1, E_2, E_3, M_f, M_c) \leq fp_{MAX}$$

(3)
\[ 0.99 \leq k_{\text{eff}}(R_f, \Delta, R_e, E_1, E_2, E_3, M_f, M_c) \leq 1.01 \]  
(4)
\[ \frac{dk_{\text{eff}}}{dV_m} > 0 \]  
(5)
\[ R_{f\min} \leq R_f \leq R_{f\max} \]  
(6)
\[ \Delta_{\text{cmin}} \leq \Delta_c \leq \Delta_{\text{cmax}} \]  
(7)
\[ R_{e\min} \leq R_e \leq R_{e\max} \]  
(8)
\[ E_{1\min} \leq E_1 \leq E_{1\max} \]  
(9)
\[ E_{2\min} \leq E_2 \leq E_{2\max} \]  
(10)
\[ E_{3\min} \leq E_3 \leq E_{3\max} \]  
(11)
\[ M_f = \{ \text{UO}_2, \text{or U-metal} \} \]  
(12)
\[ M_c = \{ \text{Zircaloy-2, Aluminum or Stainless-304} \} \]  
(13)

where \( V_m \) is the moderator volume. The \textit{Hammer} system [13] was used to make the reactor physics calculations. The fitness function to be minimized was developed in such a way that, if all constraints are satisfied, it assumes the value of the average thermal flux, \( \phi_{\text{AVE}} \). Otherwise, it is penalized proportionally to the disagreement on the constraint. The expert, according to the requirements and the priorities of the problem should set up the penalization constants \( r_i \).

\[
f = \begin{cases} 
\phi_{\text{AVE}}, & \Delta k_{\text{eff}} \leq 0.01; \; fp \leq 1.40; \; \frac{\Delta' k_{\text{eff}}}{\Delta V_m} > 0 \\
\phi_{\text{AVE}} - r_1 \Delta k_{\text{eff}}, & \Delta k_{\text{eff}} > 0.01; \; fp \leq 1.40; \; \frac{\Delta' k_{\text{eff}}}{\Delta V_m} > 0 \\
\phi_{\text{AVE}} - r_2 \Delta fp, & \Delta k_{\text{eff}} \leq 0.01; \; fp > 1.40; \; \frac{\Delta' k_{\text{eff}}}{\Delta V_m} > 0 \\
\phi_{\text{AVE}} - r_3 \Delta' k_{\text{eff}}, & \Delta k_{\text{eff}} \leq 0.01; \; fp \leq 1.40; \; \frac{\Delta' k_{\text{eff}}}{\Delta V_m} < 0 \\
\phi_{\text{AVE}} - r_4 \Delta k_{\text{eff}} - r_5 \Delta' k_{\text{eff}}, & \Delta k_{\text{eff}} > 0.01; \; fp > 1.40; \; \frac{\Delta' k_{\text{eff}}}{\Delta V_m} > 0 \\
\phi_{\text{AVE}} - r_5 \Delta fp - r_6 \Delta' k_{\text{eff}}, & \Delta k_{\text{eff}} > 0.01; \; fp > 1.40; \; \frac{\Delta' k_{\text{eff}}}{\Delta V_m} < 0 \\
\phi_{\text{AVE}} - r_7 \Delta k_{\text{eff}} - r_8 \Delta fp - r_9 \frac{\Delta' k_{\text{eff}}}{\Delta V_m}, & \Delta k_{\text{eff}} > 0.01; \; fp > 1.40; \; \frac{\Delta' k_{\text{eff}}}{\Delta V_m} < 0 \\
\phi_{\text{AVE}} - r_7 \Delta k_{\text{eff}} - r_8 \Delta fp - r_9 \frac{\Delta' k_{\text{eff}}}{\Delta V_m}, & \Delta k_{\text{eff}} > 0.01; \; fp > 1.40; \; \frac{\Delta' k_{\text{eff}}}{\Delta V_m} < 0 \\
\end{cases}
\]  
(14)

Where, \( \Delta k_{\text{eff}} = |1.0 - k_{\text{eff}}|; \; \Delta fp = fp - 1.40; \; \Delta Vm = 0.03V_m \)

4. EXPERIMENTS AND RESULTS

In order to evaluate the efficiency and robustness of the Cellular-PSO, it has been submitted to 10 experiments (see Table 2), with different random seeds. Results are compared to those obtained by a standard PSO (Table 3). For both, standard and cellular PSO, population size was 20. Constant \( c_1 \) and \( c_2 \) have been set to 2.0. The inertia weight, \( w \), decreased linearly from 0.8 to 0.2.
In this work, the Cellular PSO demonstrated to be more efficient and robust than the standard PSO, due to the neighborhood strategy used. Other strategies can be find in literature, however, the cellular model is faster, easier to implement and parallelism is more natural. Future works include the parallel implementation of the Cellular-PSO and investigation of enhancements in the model, in order to improve efficiency, robustness and speed.

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REFERENCES