NUMERICAL SIMULATION OF ULTRASONIC WAVE PROPAGATION IN ELASTICALLY ANISOTROPIC MEDIA

Victoria Cristina Cheade Jacob1,2, Reinaldo Jacques Jospin1 e Marcelo de Siqueira Queiroz Bittencourt1

1 Instituto de Engenharia Nuclear (IEN / CNEN - RJ)
Rua Hélio de Almeida 75
21941-906 Cidade Universitária, Ilha do Fundão, Rio de Janeiro, RJ
rj.jospin@ien.gov.br
bittenc@ien.gov.br

2 Bolsista de Iniciação Científica do CNPq
Universidade Veiga de Almeida
Rua Ibituruna, 108
20271-020 Tijuca, RJ
vivicheade@hotmail.com

ABSTRACT

The ultrasonic non-destructive testing of components may encounter considerable difficulties to interpret some inspections results mainly in anisotropic crystalline structures. A numerical method for the simulation of elastic wave propagation in homogeneous elastically anisotropic media, based on the general finite element approach, is used to help this interpretation. The successful modeling of elastic field associated with NDE is based on the generation of a realistic pulsed ultrasonic wave, which is launched from a piezoelectric transducer into the material under inspection. The values of elastic constants are great interest information that provide the application of equations analytical models, until small and medium complexity problems through programs of numerical analysis as finite elements and/or boundary elements. The aim of this work is the comparison between the results of numerical solution of an ultrasonic wave, which is obtained from transient excitation pulse that can be specified by either force or displacement variation across the aperture of the transducer, and the results obtained from a experiment that was realized in an aluminum block in the IEN Ultrasonic Laboratory. The wave propagation can be simulated using all the characteristics of the material used in the experiment evaluation associated to boundary conditions and from these results, the comparison can be made.

1. INTRODUCTION

The behavior characterization of ultrasonic wave which propagates in a material medium is done in a computational modeling with known acoustic characteristics. The wave propagation can be numerically simulated basically converting the differential equation in a full version that provides the discretization by the Finite Element Method. The IEN has a program of Finite Elements, MEF, which simulates the propagation of ultrasound waves in continuous media.

For this, an experiment was realized where it was possible to describe, including statistical support, the travel time of longitudinal ultrasonic wave in an aluminum block. The numerical
modeling was built in Fortran Language, where compatible variables with the physical characteristics of the real block, and the wave behavior of ultrasonic waves, were inserted.

Finally, a comparison was done between the experimental and numerical models and verified the possibility to validate this by interpreting the results.

2. EXPERIMENTAL EVALUATION

2.1. Theoretical Foundations

The ultrasonic technique uses mechanics waves which are composed by particles oscillations that propagate in the inspection media. The passage of this acoustic wave in the media makes the particles execute the oscillating movement around the equilibrium position, whose amplitude decreases with time due to the loss of energy acquired by the wave [9].

Assuming the media, under study, is elastic, in the other words, that the particles that compose it are free to oscillate in any direction, so the acoustic waves can be classified into [9]:

a) Longitudinal Waves (compression)
Are waves whose particles oscillate in the direction of wave propagation, which may be transmitted to solids, liquids and gases in the direction of wave propagation.

b) Transversal Waves (shear)
In these waves, the particles of the media vibrate in the perpendicular direction to the propagation direction, keeping plans of particles equidistant in a vertical movement. The transmission of this wave type is effective only in solids, because, in liquids and gases, the particles are practically unable to propagate due to binding characteristic of particles in these media.

2.2. Methodology

For the experimental evaluation, the travel time of ultrasonic wave is obtained by ultrasound, using longitudinal ultrasonic wave in an aluminum block.

The ultrasonic system developed and presented, FIG. 1, is composed by a computer used to capture the electronic signal present in the oscilloscope screen, through the program OpenChoice Desktop and process this signal using the program CHRONOS/IEN to obtain the travel time of ultrasonic wave (1); an oscilloscope Tektronix DPO 3032, that is used to capture and scan the ultrasonic signal to measure of ultrasonic wave travel time (2); the couplant SWC (Shear Wave Couplant) to ensure a perfect coupling (without air) between the piece inspection and transducer (3); an ultrasound Panametrics, EPOCH4 PLUS model (4); an aluminum block with dimensions 70.00 x 70.02 x 70.01 [mm] (5); the longitudinal wave transducer Panametrics, A109S model, 5MHz frequency and 0.5” diameter, which works as transmitter and receiver and is coupled in the material inspection (6); Digital Thermometer, Text brand, 177 - T3 model, which lets to control the local temperature and the aluminum block to be studied (7).
First of all, an area was marked on each face of the block to be measured and the materials to be used are placed. The transducer is coupled in the aluminum block to acquire the signals at the three areas of the block. The capture of ultrasonic signals is done by keeping the transducer coupled in the block until acquire the thirty signals established for the study. After data acquisition, a data treatment was realized in the Chronos program (cross-correlation) [1], getting travel times of ultrasonic wave.
2.3. Results

The results obtained in the experimental evaluation of the aluminum block, for the travel time of the longitudinal ultrasonic wave, are showed in Table 1, for its propagation in the three faces of the aluminum block.

Table 1: Travel Time of Longitudinal Ultrasonic Wave in an Aluminum Block.

<table>
<thead>
<tr>
<th>TRAVEL TIME OF LONGITUDINAL ULTRASONIC WAVE (ns)</th>
<th>FACE 1</th>
<th>FACE 2</th>
<th>FACE 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>22130.0</td>
<td>22043.2</td>
<td>21892.8</td>
<td></td>
</tr>
<tr>
<td>22124.0</td>
<td>22043.6</td>
<td>21892.4</td>
<td></td>
</tr>
<tr>
<td>21888.8</td>
<td>22044.0</td>
<td>21892.0</td>
<td></td>
</tr>
<tr>
<td>21888.4</td>
<td>22040.4</td>
<td>21892.4</td>
<td></td>
</tr>
<tr>
<td>21888.0</td>
<td>22039.2</td>
<td>21892.0</td>
<td></td>
</tr>
<tr>
<td>22114.8</td>
<td>22038.4</td>
<td>21891.6</td>
<td></td>
</tr>
<tr>
<td>22115.6</td>
<td>22038.4</td>
<td>22092.4</td>
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</tr>
<tr>
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<tr>
<td>22114.8</td>
<td>22036.4</td>
<td>22091.6</td>
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<tr>
<td>21888.8</td>
<td>22035.6</td>
<td>22090.8</td>
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<tr>
<td>21889.6</td>
<td>22035.2</td>
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<tr>
<td>22130.0</td>
<td>22034.4</td>
<td>21892.8</td>
<td></td>
</tr>
</tbody>
</table>

Mean: 22021.8  22038.3  22011.5
Standard Deviation: 111.309  3.219  99.0928
According to experiments that were realized before in the Ultrasound Laboratory, the following relationship was obtained (1), which is in agreement with theory:

\[ v_T = 0.5v_L \quad (1) \]

From the knowledge of the material density and relations presented below for longitudinal and shear velocities:

\[ v_L = \frac{\lambda + 2\mu}{\rho} \quad v_T = \frac{\mu}{\rho} \quad (2) \]

Where the material properties of the media (\( \lambda \) and \( \mu \)) are defined by relations below:

\[ \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad \mu = G = \frac{E}{2(1+\nu)} \quad (3) \]

is possible to obtain the values of the constants: elastic modulus, the shear modulus and Poisson's ratio. In Table 2 below, the values obtained are showed for these constants and for longitudinal velocity, and travel time of longitudinal ultrasonic wave.

### Table 2: Results from Experimental Evaluation

<table>
<thead>
<tr>
<th></th>
<th>FACE I</th>
<th>FACE II</th>
<th>FACE III</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E )  [ GPa ]</td>
<td>67.8166</td>
<td>67.7539</td>
<td>67.8992</td>
<td>67.8232</td>
</tr>
<tr>
<td>( G )  [ GPa ]</td>
<td>25.4311</td>
<td>25.4077</td>
<td>25.4622</td>
<td>25.4334</td>
</tr>
<tr>
<td>( v_L ) [ m/s x 10^3 ]</td>
<td>6.3573</td>
<td>6.3544</td>
<td>6.3612</td>
<td>6.3576</td>
</tr>
<tr>
<td>( v )</td>
<td>0.3350</td>
<td>0.3369</td>
<td>0.3344</td>
<td>0.3354</td>
</tr>
<tr>
<td>( t_L ) [ns]</td>
<td>22021.8</td>
<td>22038.3</td>
<td>22011.5</td>
<td>22023.9</td>
</tr>
<tr>
<td>( \rho ) [ kg/m^3 x 10^3 ]</td>
<td>2.5170</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Where:

- \( E \) - Young's Module
- \( G \) - Shear Module
- \( v_L \) - Longitudinal Velocity
- \( v \) - Poisson's Ratio
- \( t_L \) - Travel Time of Longitudinal Ultrasonic Wave
- \( \rho \) - Density of the Aluminum Block
3. NUMERICAL SIMULATION

3.1. Theoretical Foundations

The numerical simulation of wave propagation in continuous media [6], [4] is based on the differential equation defined by the equation (4):

\[
(\lambda + 2\mu)\nabla(\nabla \cdot u) - \mu \nabla \times (\nabla \times u) = \rho \ddot{u}
\]  

(4)

where \( u(x, y) \) is the displacement of the plate at point \( (x, y) \) and \( \ddot{u}(x, y) \) is the acceleration of this point.

Integrating this equation in the domain and using the Galerkin method, we can re-write it as shown in (5) with the boundary conditions defined in (6) and (7):

\[
\int_\Omega [T \cdot \nabla w - \rho \ddot{u} \cdot w] d\Omega = 0
\]  

(5)

\( T \cdot n = t \) \text{ in } \Gamma_t

(6)

\( u = \ddot{u} \) \text{ in } \Gamma_u

(7)

where \( T \) is the stress tensor, which acts in the body, \( w(x, y) \) is a weight function and where the contour of the body is denoted by \( \Gamma = \Gamma_u \cup \Gamma_t \) satisfying the relation \( \Gamma_u \cap \Gamma_t = \emptyset \).

This integral form is applicable for a discretization using the Finite Element Method.

To obtain an approximation of the integral formulation, a discretization of the domain \( x(x,y,z) \) and the displacement field \( u(x,y,z) \) is used by the Finite Element Method. The discretization uses a triangular finite element which is associated to a quadratic interpolation function associated with each node in the element and defined in sub-domain element. The combination of all elements allows obtaining an interpolation function for the whole domain. This discretization applied to the integral formulation allows obtaining a system of equations as defined by the equation (8).

\[
M^e \ddot{u}^e + K^e u^e = R^e
\]  

(8)

The propagation of ultrasonic waves in continuous media is based on transducer pre-defined function, in the case of L-wave represented by the function presented in equation (9).

\[
u(x,y,t) = \delta(x)w(y)f(t)
\]  

(9)

The exciter function \( f(t) \) is a cosine function acting for 3 microseconds and then annulling, as shown in equation (10) e (11).

\[
f(t) = (1 - \cos \omega_0 t) \cos \omega_0 t \quad \text{for} \quad 0 \leq t \leq 3\mu s
\]  

(10)

\[
f(t) = 0 \quad \text{for} \quad t > 3\mu s
\]  

(11)
A graphical representation of this excitation is given, FIG. 2.

![Figure 2: Transducer Excitation](image)

3.2. Methodology

In the numerical simulation case, the pre-processor that defines geometry, materials, boundary conditions and the forces or displacements caused by the transducer is done using a commercial program named GID. This program serves as the graphical interface to the MEF program for the pre and the pos-processing allowing the preview of the wave movement in the continuous medium. The simulation of ultrasonic wave in an aluminum block was done to get a comparison of the results obtained in the experiment evaluation. In this way, the experimental evaluation may be used to validate the numerical evaluation [3], [5], [2], [1].

The graphical representation of simulation in an aluminum plate is given, FIG. 3.

![Figure 3: L-wave propagation in an aluminum plate.](image)
3.3. Results

The results obtained in numerical simulation of the aluminum block, which is discretized by 1536 quadratic triangular elements (T6) and uses a time step of $1 \times 10^{-8} [s]$, are showed in Table 3.

### Table 3: Numerical Simulation

<table>
<thead>
<tr>
<th>Given</th>
<th>Analytical Solution</th>
<th>MEF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E = 70 \times 10^9 [Pa]$</td>
<td>$v = 0.33$</td>
<td>$\rho = 2.71 \times 10^3 [kg/m^3]$</td>
</tr>
<tr>
<td>$t_L$</td>
<td>$-2$</td>
<td>$-2$</td>
</tr>
<tr>
<td>$v_L$</td>
<td>$6.1864 \times 10^3 [m/s]$</td>
<td>$5.6444 \times 10^3 [m/s]$</td>
</tr>
<tr>
<td>$v_T$</td>
<td>$3.1162 \times 10^3 [m/s]$</td>
<td>$3.4324 \times 10^3 [m/s]$</td>
</tr>
</tbody>
</table>

4. COMPARING RESULTS

4.1. Experimental Evaluation vs. Literature

According to CALLISTER (2012) [8], Young's Module, the Shear Module and Poisson’s Ratio, in metallic alloys, have the following values:

$$E = 69.0 [GPa]$$
$$G = 25.0 [GPa]$$
$$v = 0.33$$

As can be seen, experimental results (Table 2) are near to Literature Data. So, it can be observed there is coherence of data obtained from experimental evaluation.

4.2. Experimental Evaluation vs. Numerical Simulation

Comparing the experimental results (Table 2) and numerical simulation (Table 3), it can be seen that the numerical simulation can reproduce, with errors approximation around 5%, the experiment results obtained in Ultrasound Laboratory, using as comparison $v_L$ (longitudinal velocity), since the block has similar properties in each direction. Differences between experimental and simulation results can have several sources due to approximations, assumptions and / or hypotheses. Another possible difference in results is the presence of residual stresses resulting from the manufacturing process, in which case it’s known that there are differences in propagation velocities, but whose objective was not presented of this analysis.

5. CONCLUSION

It was concluded that experimental evaluation studied in aluminum block can be used as a good approximation of the numerical simulation, observed that longitudinal velocity found is experimentally compatible with the velocity used in the Laboratory.
ACKNOWLEDGMENTS

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